A NOTE ON NORDHAUS-GADDUM-TYPE INEQUALITIES FOR THE AUTOMORPHIC $\mathcal{H}$-CHROMATIC INDEX OF GRAPHS

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Abstract

The automorphic $H$-chromatic index of a graph $G$ is the minimum integer $m$ for which $G$ has a proper edge-coloring with $m$ colors which is preserved by a given automorphism group $H$ of $G$. We consider the sum and the product of the automorphic $H$-chromatic index of a graph and its complement. We prove upper and lower bounds in terms of the order of the graph when $H$ is chosen to be either a cyclic group of prime order or a group of order four.

1 Introduction

All graphs under consideration are simple. For graph terminology and notation we refer to [6]. Let $G = (V, E)$ be a graph of order $n$ with vertex set $V$ and edge set $E$. The complement $\overline{G}$ of a graph $G$ is the graph whose vertex set is that of $G$ and in which two vertices are adjacent if and only if they are not adjacent in $G$. Let $k \geq 2$ be an integer. Following [7] we define a $k$-decomposition of a graph $G_0$ as a family $(G_1, G_2, \ldots, G_k)$ of spanning subgraphs of $G_0$ such that each edge of $G_0$ is contained in exactly one member of $(G_1, G_2, \ldots, G_k)$, see also [3]. We shall occasionally refer to the subgraphs $G_1, G_2, \ldots, G_k$ as being the “blocks” of the $k$-decomposition.

The following two problems can be formulated for an arbitrary graph parameter $P$:

1. finding upper and lower bounds of the set

$$\{P(G_1) + \cdots + P(G_k) : (G_1, G_2, \ldots, G_k) \text{ is a } k\text{-decomposition of } G_0\};$$

2. finding upper and lower bounds of the set

$$\{P(G_1) \cdot P(G_2) \cdots P(G_k) : (G_1, G_2, \ldots, G_k) \text{ is a } k\text{-decomposition of } G_0\}.$$ 

The study of the above problems started in 1956 with the paper by Nordhaus and Gaddum [10] in the particular case $k = 2$, $G_0$ the complete graph $K_n$ of order $n$ and $P = \chi$ the chromatic number. Nordhaus and Gaddum gave answers to problems (1) and (2) in terms of the order $n$ of $G_0 = K_n$. Only 10 years later Vizing in [11] solved the same problems for another graph parameter, namely the chromatic index $P = \chi'$.

**Theorem 1.1.** [11] For an arbitrary graph $G$ of order $n$ the following inequalities hold:

$$2 \left\lfloor \frac{n + 1}{2} \right\rfloor - 1 \leq \chi'(G) + \chi'(\overline{G}) \leq n + 2 \left\lfloor \frac{n - 2}{2} \right\rfloor,$$